

Photoproduction of the πN pair on nuclei and isobar configurations

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The model of the πN pair photoproduction on the nuclei at the high momentum transfer was presented. The amplitude of the $A(\gamma, \pi N)B$ process was received by means of the extended impulse approximation, according to which nucleus, besides the nucleon components, includes also the delta-isobar degrees of freedom. One-particle operator of the transition was defined by amplitudes of the elementary $\gamma N \rightarrow \pi N$ and $\gamma \Delta \rightarrow \pi N$ processes. The direct and exchange mechanisms of the reaction were analysed. The numerical estimations of the differential cross section of the $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ and $^{12}\text{C}(\gamma, \pi^+ p)^{11}\text{Be}$ reactions were given.

1 Introduction

The photoproduction of pion on the nucleus, which is accompanied by emission of nucleons, is useful for the study of such problem of the nuclear physics as "Δ-isobar in nuclei". Usually three substantially different facets of the delta-isobar are considered in the nucleus. These facets differ in the isobar-production mechanism and isobar state. Quasifree isobar production in a "free" state in the scattering of high-energy particles on nuclei has received the most comprehensive study. In this case, the isobar is produced nearly on-mass-shell, because the energy-momentum transfer to a bound intranuclear nucleon is quite high. Such an isobar propagates in the nucleus involved, interacting with the closest nucleons, and decays with a high probability to a pion and a nucleon or undergoes the transition $\Delta N \rightarrow NN$ accompanied by the knockout of two nucleons.

Another facet of Δ-nucleus physics is associated with isobar configurations in the ground state of the nuclei. At intermediate distances the most of the attraction between two nucleons of nuclei comes from the exchange of two pions, between which one or two Δs can be created. Thus, as a result of nucleon collisions the creation of the virtual delta-isobars is possible. These delta-isobars are produced far from the mass shell and therefore cannot undergo the decay $\Delta \rightarrow N\pi$, but they can transit into a free state upon the transfer of the required 4-momentum to them from a high-energy particle. The excitations of the bound nucleons are most intensive at the large momentum transfer, therefore the virtual delta-isobars are connected with the high-momentum components of nuclear wave function. In the framework of the non-relativistic semi-phenomenological model the virtual isobar has led to the so-called isobar configuration in nuclei [1, 2]. In this model the conventional wave function consisting of nucleons is supplemented by exotic components in which one or several nucleons are internally excited, i.e. are baryon resonances or isobars.

The third facet, which has to be studied adequately, is associated with a hypothetical quasibound delta-nucleus state of a nucleus. In many respects delta-nucleon interaction is similar to nucleon-nucleon interaction, which is attractive [3]. Therefore, it can be hypothesized that under favorable conditions such that the momentum of the product or knock-on isobar is small in relation to the momentum of nucleons bound in the nucleus involved, the delta isobar and the residual nucleus may form a highly excited bound state (Δ -nucleus). This is not an ordinary bound nuclear state, since it is unstable with respect to the emission of a pion or a pion-nucleon pair. Moreover, the lifetime of a free isobar is substantially shorter than the time required for the formation of a normal collective nuclear state, according to [4], therefore, we will refer to the states in question as quasibound states.

The possible existence of bound and resonance delta-nucleon and delta-nucleus states was widely discussed in [5–9]. From the experimental point of view, conclusion of the work [4] that a quasibound delta-nucleon state of isospin $T = 2$ and spin-parity $J^P = 2^+$ may exist is the most appealing. The binding energy of this state is estimated at 10 to 40 MeV depending on the approximations used in relevant calculations. Last time this problem was considered in the works [10, 11], where the results of the experiment at Tomsk synchrotron were discussed. The cross section for the $C^{12}(\gamma, \pi^- p)$ reaction was measured in the $\Delta(1232)$ -resonance region. This experiment possibly indicates the existence of quasibound isobar-nucleus states. The analogical conclusions were made in the works [12, 13], in which the author considered data from three experiments performed at the linear accelerator in Saclay [14], at Tomsk synchrotron [15] and at MAMI accelerator in Mainz [16] and devoted to exploring the photoproduction of single pions on light nuclei that is accompanied by nucleon emission in the $\Delta(1232)$ -resonance region.

The conclusions of the works [10–13] were based on comparison of the experimental data of the $A(\gamma, \pi N)$ reaction with the theoretical predictions obtained in the frame of the model, based on the hypotheses of the Δ -nucleus existence. This reaction mechanism is manifested in the region of high momentum transfers to the residual nuclear system. However, in the same kinematical region other possible concomitant mechanisms of the reaction also occur. Particularly, manifestations of the isobar configurations in the nucleus ground state and meson-exchange currents are possible. An analysis of these reaction mechanisms is needed for testing conclusions drawn in [10–13] about the existence of quasibound isobar-nucleus states and for interpretation of $(\gamma, \pi^+ p)$ and $(\gamma, \pi^- n)$ reaction data.

In this work we study the influence of isobar configurations on photoproduction of pion-nucleon pairs on light nuclei with closed shells. The basic ingredients of the reaction model presented are nucleus density matrices, taking into account nucleon and isobar degrees of freedom, and single-particle operators of $\gamma N \rightarrow \pi N$ and $\gamma \Delta \rightarrow \pi N$ transitions. Direct and exchange reaction mechanisms are considered. Using this model, we calculate the contribution of isobar configurations to the cross section of the $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ reaction in the area where the existence of quasibound isobar-nucleus states is expected and estimate numerically the cross section of the $^{12}\text{C}(\gamma, \pi^+ p)^{11}\text{Be}$ reaction in the region of high momentum transfer.

2 Amplitude of the $A(\gamma, \pi N)B$ reaction

The matrix element of the S -matrix between the initial state i and the final state f , describing the reaction of the pion photoproduction on nucleus A is accompanied by the emission of

nucleon N and the formation of a residual nucleus B can be represented in the form

$$\langle f | S | i \rangle = -2\pi \delta(E_\gamma + E_T - E_\pi - E_N - E_R) \frac{T_{fi}}{(2E_\gamma 2E_\pi)^{1/2}}, \quad (2.1)$$

where E_γ , E_T , E_π , E_N and E_R are energies of the photon, the initial nucleus A , the pion, the nucleon and the residual nucleus B ; T_{fi} is the transition matrix element of the $A(\gamma, \pi N)B$ reaction.

For calculation of the matrix element T_{fi} we will use the approach developed in the works of Arenhovel et. al. [2, 17, 18] for study the isobar configurations of the ground states of the light nuclei. Here, we apply this formalism for the description of the nuclear reactions.

According to [2, 17, 18], baryons bound in the nucleus, in addition to the space \mathbf{r} , spin s , and isospin t coordinates ($\mathbf{r}, s, t \equiv x$), are characterized also by the intrinsic coordinate m ($x, m \equiv X$). An eigenfunction $\Psi_\beta(X_1, \dots, X_A)$ of hamiltonian H of system of A particles with eigenvalue E_β is a superposition of the wave functions concerned with different intrinsic configurations

$$\Psi_\beta(X_1, \dots, X_A) = \sum_n \Psi_\beta^n(X_1, \dots, X_A),$$

$$\Psi_\beta^n(X_1, \dots, X_A) = A_n \phi_n(m_1, \dots, m_A) \Psi_\beta^n(x_1, \dots, x_A).$$

Here $\phi_n(m_1, \dots, m_A)$ is the intrinsic wave function of A particles. The index $n \equiv n_1, \dots, n_A$ characterize the intrinsic state of the particles. For instance, the state index describing the intrinsic configuration of the nucleons system is written as $n = N_1, N_2, \dots, N_A$; if the first particle is in the state of isobar, but the rest are nucleons, the intrinsic state index is written as $n = \Delta_1, N_2, \dots, N_A$. By definition, $\Psi_\beta^n(x_1, \dots, x_A)$ is the wave function describing the state of A particles with quantum numbers $\beta \equiv \beta_1, \dots, \beta_A$ in the usual space, spin and isospin spaces, and with quantum numbers $n = n_1, \dots, n_A$ in the intrinsic space. The wave function $\Psi_\beta^n(x_1, \dots, x_A)$ should be antisymmetric for particles in the same intrinsic state. The remaining antisymmetrization for particles in different intrinsic states is done by the operator A_n .

In the frame of this approach we define the matrix element T_{fi} of the $A(\gamma, \pi N)B$ reaction in configuration space, which, in addition to the usual space, spin, isospin coordinates, also includes the intrinsic coordinates, as

$$T_{fi} = A \int d(X'_1, X_1, \dots, X_A) \Psi_F^*(X'_1, X_2, \dots, X_A) \times \\ \times \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \Psi_T(X_1, \dots, X_A).$$

Here the integral sign denotes the integration over the space variables and summation over the spin, isospin and intrinsic variables; $\Psi_F(X_1, \dots, X_A)$ is the antisymmetrized wave function of the final nuclear system F including residual nucleus B and nucleon N in the free state; $\Psi_T(X_1, \dots, X_A)$ is the antisymmetrized wave function of nucleus A ; $t_{\gamma\pi}$ is the single-particle operator of pion photoproduction on a free baryon.

Let us present the wave function Ψ_F of the final nuclear system as the antisymmetrized product of the wave function $\varphi_{\mathbf{p}_n}$ of the free nucleon with the momentum \mathbf{p}_n and the wave function Ψ_f of nucleus B in the state f :

$$\Psi_F(X_1, \dots, X_A) = A_{1;2\dots A}^p \varphi_{\mathbf{p}_n}(X_1) \Psi_f(X_2, \dots, X_A),$$

where $A_{1;2\dots A}^p$ is the antisymmetrization operator. Then, we obtain for the T -matrix

$$T_{fi} = T_d - T_e.$$

Here the direct amplitude T_d is

$$T_d = \sqrt{A} \int d(X'_1, X_1, \dots, X_A) \varphi_{\mathbf{p}_n}^*(X'_1) \Psi_f^*(X_2, \dots, X_A) \times \\ \times \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \Psi_T(X_1, \dots, X_A),$$

in which the "active" particle with number 1 interacts with the photon and changes in state from bound to free; the exchange amplitude T_e is

$$T_e = \sqrt{A}(A-1) \int d(X'_1, X_1, \dots, X_A) \varphi_{\mathbf{p}_n}^*(X_2) \Psi_f^*(X'_1, X_3, \dots, X_A) \times \\ \times \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \Psi_T(X_1, \dots, X_A), \quad (2.2)$$

in which the "active" particle remains in the bound state after the interaction with the photon.

Let us now write the quadrate of the modulus of the amplitude T_{fi}

$$T_{fi} T_{fi}^* = T_d T_d^* + T_e T_e^* - T_d T_e^* - T_e T_d^*$$

The quadrate of the modulus of the direct amplitude T_d is

$$T_d T_d^* = A \int d(X'_1, X_1, \tilde{X}'_1, \tilde{X}_1) \varphi_{\mathbf{p}_n}^*(X'_1) \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \psi_{A-1,A}(X_1) \times \\ \times \psi_{A-1,A}^*(\tilde{X}_1) \langle \tilde{X}_1 | t_{\gamma\pi}^+ | \tilde{X}'_1 \rangle \varphi_{\mathbf{p}_n}(\tilde{X}'_1),$$

where

$$\psi_{A-1,A}(X_1) = \int d(X_2, \dots, X_A) \Psi_f^*(X_2, \dots, X_A) \Psi_T(X_1, \dots, X_A)$$

is the overlap function.

The differential cross section of the reaction $A(\gamma, \pi N)B$, summed over all the final states of the nucleon and the residual nucleus will be considered. Let us accept the condition of the completeness of final states

$$\sum_f \Psi_f^*(X_2, \dots, X_A) \Psi_f(\tilde{X}_2, \dots, \tilde{X}_A) = \delta(X_2 - \tilde{X}_2) \dots \delta(X_A - \tilde{X}_A),$$

where sum is taken over all the final states of the residual nucleus. In this case the expression for the quadrate of the modulus of the amplitude T_d is

$$\sum_f T_d T_d^* = A \int d(X'_1, X_1, \tilde{X}'_1, \tilde{X}_1) \varphi_{\mathbf{p}_n}^*(X'_1) \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \times \\ \times \rho(X_1; \tilde{X}_1) \langle \tilde{X}_1 | t_{\gamma\pi}^+ | \tilde{X}'_1 \rangle \varphi_{\mathbf{p}_n}(\tilde{X}'_1),$$

where

$$\rho(X_1; \tilde{X}_1) = \int d(X_2, \dots, X_A) \Psi_T(X_1, \dots, X_A) \Psi_T^*(\tilde{X}_1, X_2, \dots, X_A)$$

is the one-body density matrix.

We will now consider the exchange amplitude T_e . In the case of the exchange mechanism of the charge pion photoproduction, the "active" nucleon remaining in the bound state can transit to the states, which are above or low the Fermi level. The last vacant levels were produced as the result of the process of the Δ -isobar production by means of the transition $NN \rightarrow \Delta N$. Also, the "active" nucleon can transit to the vacant level arose as the result of the virtual decay $A \rightarrow (A-1) + N$. In the case of the neutral pion photoproduction the exchange amplitude contains additionally the transitions $\gamma N \rightarrow N\pi$ without change of the nucleon state.

In the case, if the "active" nucleon goes to the state, which is above the Fermi level, the wave function of the residual nucleus may be written as

$$\Psi_f(X'_1, X_3, \dots, X_A) = A_{u;1\dots A \neq k\dots l}^S \Psi_{\beta_u}(X'_1) \Psi_{(\beta_k\dots\beta_l)^{-1}}(X_3, \dots, X_A), \quad (2.3)$$

where $\Psi_\beta(X)$ is one-particle wave function of the nucleon bound in nuclei, β_u is the index of the nucleon state which is above the Fermi level, $(\beta_k\dots\beta_l)^{-1}$ is the hole state of the bound system of barions with numbers 3, ..., A, the antisymmetrization operator $A_{u;1\dots A \neq k\dots l}^S$ rearranges the indices of the nucleon states.

As all nucleons of the wave function Ψ_T are lower than the Fermi level, the nonzero contribution of the exchange amplitude arises from the first summand

$$\Psi_{\beta_u}(X'_1) \Psi_{(\beta_k\dots\beta_l)^{-1}}(X_3, \dots, X_A)$$

of the expression (2.3). As a result, the quadrate of the modulus of the exchange amplitude is

$$\begin{aligned} T_e T_e^* &= A(A-1) \int d(X'_1, X_1, X_2, \tilde{X}'_1, \tilde{X}_1, \tilde{X}_2) \varphi_{\mathbf{p}_n}^*(X_2) \Psi_{\beta_u}^*(X'_1) \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \\ &\times \psi_{A-2,A}(X_1, X_2) \psi_{A-2,A}^*(\tilde{X}_1, \tilde{X}_2) \langle \tilde{X}_1 | t_{\gamma\pi}^+ | \tilde{X}'_1 \rangle \Psi_{\beta_u}(\tilde{X}'_1) \varphi_{\mathbf{p}_n}(\tilde{X}_2), \end{aligned}$$

where

$$\psi_{A-2,A}(X_1, X_2) = \int d(X_3, \dots, X_A) \Psi_{(\beta_k\dots\beta_l)^{-1}}^*(X_3, \dots, X_A) \Psi_T(X_1, \dots, X_A).$$

If the set of the states $(\beta_k\dots\beta_l)^{-1}$ is full, then

$$\begin{aligned} \sum_f T_e T_e^* &= A(A-1) \sum_u \int d(X'_1, X_1, X_2, \tilde{X}'_1, \tilde{X}_1, \tilde{X}_2) \varphi_{\mathbf{p}_n}^*(X_2) \Psi_{\beta_u}^*(X'_1) \times \\ &\times \langle X'_1 | t_{\gamma\pi} | X_1 \rangle \rho(X_1, X_2; \tilde{X}_1, \tilde{X}_2) \langle \tilde{X}_1 | t_{\gamma\pi}^+ | \tilde{X}'_1 \rangle \Psi_{\beta_u}(\tilde{X}'_1) \varphi_{\mathbf{p}_n}(\tilde{X}_2). \end{aligned}$$

Here

$$\rho(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \int d(X_3, \dots, X_A) \Psi_T(X_1, X_2, \dots, X_A) \Psi_T^*(\tilde{X}_1, \tilde{X}_2, \dots, X_A)$$

is the two-body density matrix.

We will neglect the contribution of the products $T_d T_e^*$ and $T_e T_d^*$, as the kinematical regions, in which the basic contribution of the direct and exchange amplitudes in cross section differ considerably.

3 Nucleus wave function

The nucleus wave functions $\Psi_\beta(X_1, \dots, X_A)$ satisfy the following Schrödinger equation

$$(H - E_\beta) \Psi_\beta(X_1, \dots, X_A) = 0, \quad (3.1)$$

where the hamiltonian of system H acts on spatial, spin, isospin, and intrinsic coordinates. According to [18] the hamiltonian H have the form

$$H = \sum_{i=1}^A T_i + I_i + \sum_{i<j} V_{ij} = H_0 + V.$$

Here T_i is the kinetic energy operator of i -particle, I_i is the part connected with the intrinsic degrees of freedom, V_{ij} is the two-particle interaction. The operators T and V , unlike those of standard nuclear physics, also depend on the intrinsic degrees of freedom. The operators T and I are diagonal by the intrinsic degrees of freedom.

The wave function of the nucleus in eq. (3.1) may be written as

$$\Psi(X_1, \dots, X_A) = \Psi_N(X_1, \dots, X_A) + \Psi_\Delta(X_1, \dots, X_A), \quad (3.2)$$

where

$$\Psi_N(X_1, \dots, X_A) = \phi_N(m_1, \dots, m_A) \Psi_\beta^N(x_1, \dots, x_A)$$

is the wave function of the nucleus in the state, when all particles of the nucleus are nucleons; intrinsic state index $N \equiv N_1, N_2, \dots, N_A$;

$$\Psi_\Delta(X_1, \dots, X_A) = \sum_{\Delta} A_{\Delta} \phi_{\Delta}(m_1, \dots, m_A) \Psi_{\beta}^{\Delta}(x_1, \dots, x_A)$$

is the wave function of the nucleus, which includes the states with one isobar $\Delta = \Delta_1, N_2, \dots, N_A$, two isobars $\Delta = \Delta_1, \Delta_2, N_3, \dots, N_A$ and etc. The wave functions Ψ_N and Ψ_Δ are normalized correspondingly by N_N and N_Δ .

The wave functions $\Psi_\beta^\Delta(X_1, \dots, X_A)$ of the Δ -configuration satisfy the following equation

$$(H - E_\beta) \Psi_\beta^\Delta(X_1, \dots, X_A) = - \sum_{n' \neq \Delta} V \Psi_\beta^{n'}(X_1, \dots, X_A). \quad (3.3)$$

Since those configurations, in which one or several nucleons are in an intrinsically excited state, are expected to be small because of the large excitation energy, for this equation one can find an approximation solution in a perturbative approach, according to which one can leave only $n' = N = N_1, N_2, \dots, N_A$ configuration on the right-hand side of eq. (3.3). Then, in this approximation we will have the equation

$$(H - E_\beta) \Psi_\beta^\Delta(X_1, \dots, X_A) = -V \Psi_\beta^N(X_1, \dots, X_A).$$

In our model we will take into account only the dominant one- Δ configuration. Assuming that only two nucleons are involved in the excitation of the nucleon internal degrees of freedom, wave function Ψ_Δ of one- Δ configuration can be written as the superposition of the products of the wave function of ΔN system, which includes an isobar and the second

nucleon (the participant of the transition $NN \rightarrow \Delta N$) and the wave function describing the state of other $A-2$ nucleons,

$$\Psi_{\Delta}(X_1, \dots, X_A) = A_{12;3,\dots,A}^P \sum_{ij} \Psi_{[\beta_i\beta_j]}^{\Delta N}(X_1, X_2) \Psi_{(\beta_i\beta_j)^{-1}}^N(X_3, \dots, X_A). \quad (3.4)$$

Here

$$A_{12;3,\dots,A}^P = \sqrt{\frac{2}{A(A-1)}} \left(1 - \sum_{i=3}^A (P_{1i}^P + P_{2i}^P) + \sum_{i=3}^{A-1} \sum_{j=i+1}^A P_{1i}^P P_{2j}^P \right)$$

is the antisymmetrization operator, the operator P_{ik}^P transposes the i -th and k -th nucleons,

$$\Psi_{[\beta_i\beta_j]}^{\Delta N}(X_1, X_2) = A_{1;2}^P \phi_{\Delta N}(m_1, m_2) \Psi_{[\beta_i\beta_j]}^{\Delta N}(x_1, x_2), \quad A_{1;2}^P = \frac{1}{\sqrt{2}}(1 - P_{12}^P), \quad (3.5)$$

$$\Psi_{(\beta_i\beta_j)^{-1}}^N(X_3, \dots, X_A) = \phi_N(m_3, \dots, m_A) \Psi_{(\beta_i\beta_j)^{-1}}^N(x_3, \dots, x_A).$$

The wave function of the space \mathbf{r} , spin s , and isospin t coordinates of A particles with quantum number Δ satisfy the following Schrödinger equation

$$(\phi_{\Delta}, (H - E_{\beta}) A_{\Delta} \phi_{\Delta}) \Psi_{\beta}^{\Delta}(x_1, \dots, x_A) = -(\phi_{\Delta}, V \phi_N) \Psi_{\beta}^N(x_1, \dots, x_A), \quad (3.6)$$

If we neglect the interaction between isobars and nucleons and between isobars themselves and take into account (3.4) on the left-hand side of the eq. (3.6), then the wave function $\Psi_{[\beta_i\beta_j]}^{\Delta N}$ of ΔN system satisfies the equation

$$\begin{aligned} & \sqrt{\frac{2}{A(A-1)}} (\phi_{\Delta}, (T_1 + T_2 + M_1 - M_2 - E_{\beta} - E_{(\beta_i\beta_j)^{-1}}) \phi_{\Delta}) \Psi_{[\beta_i\beta_j]}^{\Delta N}(x_1, x_2) = \\ & - \int d(x_3, \dots, x_A) \Psi_{(\beta_i\beta_j)^{-1}}^{N*}(x_3, \dots, x_A) (\phi_{\Delta}, V \phi_N) \Psi_{\beta}^N(x_1, \dots, x_A), \end{aligned}$$

where T_1 , T_2 and M_1 , M_2 are the kinetic energy operators and masses of Δ -isobar and nucleon; $(\phi_{\Delta}, V \phi_N)$ is the transition potential.

For this equation the analytic form of the wave function $\Psi_{[\beta_i\beta_j]}^{\Delta N}$ of the ΔN system in the nuclei with closed shells derived for the oscillator shell model of the nuclei with ls -coupling and one boson exchange transition potential was given in [18]. The wave function $\Psi_{[\beta_i\beta_j]}^{\Delta N}$ for the shell model with jj -coupling and the same transition potential was obtained in the work [19].

4 One-particle density matrix

According to the form of the wave function (3.2), the density matrix may be written as

$$\rho = \rho_{NN} + \rho_{\Delta\Delta} + \rho_{N\Delta} + \rho_{\Delta N}. \quad (4.1)$$

We shall analyze only diagonal components of the density matrix ρ_{NN} and $\rho_{\Delta\Delta}$. Because of the orthogonality of one-particle states, the contribution from the non-diagonal components of the density matrix to the quadrate of the modulus of the transition amplitude is expected to be small or zero.

We shall consider the first term of one-particle density matrix (4.1)

$$\rho_{NN}(X_1; \tilde{X}_1) = \int d(X_2, \dots, X_A) \Psi_N(X_1, \dots, X_A) \Psi_N^*(\tilde{X}_1, X_2, \dots, X_A).$$

The wave function $\Psi_N(X_1, X_2, \dots, X_A)$ can be written as follows

$$\Psi_N(X_1, X_2, \dots, X_A) = \sqrt{N_N} A_{1;2\dots A}^S \Psi_{\beta_1}(X_1) \Psi_{\beta_1^{-1}}^N(X_2, \dots, X_A),$$

where the wave functions

$$\Psi_{\beta_i}(X_1) = \phi_N(m_1) \Psi_{\beta_i}(x_1) \quad (4.2)$$

and $\Psi_{\beta_1^{-1}}^N(X_2, \dots, X_A)$ are normalized by 1. As a result we shall get

$$\rho_{NN}(X_1, \tilde{X}_1) = \phi_N(m_1) \left[\frac{N_N}{A} \rho_{NN}(x_1, \tilde{x}_1) \right] \phi_N^*(\tilde{m}_1), \quad (4.3)$$

where

$$\rho_{NN}(x_1, \tilde{x}_1) = \sum_{i=1}^A \Psi_{\beta_i}(x_1) \Psi_{\beta_i}^*(\tilde{x}_1),$$

The one-particle density matrix is used in the expression for the quadrate of the modulus of the direct amplitude T_d , which has the following structure: the operator $t_{\gamma\pi}$ acts on the particle with the coordinate X_1 , which moves over to the free nucleon state, and the system of the particles with numbers 2, ..., A is a "spectator". In eq. (4.3) particle "1" is a nucleon. Therefore, the summand ρ_{NN} corresponds to the quasifree mechanism of the reaction, which is illustrated by the diagram in Fig. 1a. The pion production occurs at interaction of the photon with the nucleon of the nucleus as a result of the process $\gamma N \rightarrow N'\pi$. A spectator is a system of $A-1$ barions, forming the residual nucleus, when all particles are the nucleons.

The second summand of one-particle density matrix (4.1) is

$$\rho_{\Delta\Delta}(X_1; \tilde{X}_1) = \int d(X_2, \dots, X_A) \Psi_{\Delta}(X_1, X_2, \dots, X_A) \Psi_{\Delta}^*(\tilde{X}_1, X_2, \dots, X_A). \quad (4.4)$$

Substituting in (4.4) the expression (3.4) for the wave function Ψ_{Δ} , we shall get

$$\begin{aligned} \rho_{\Delta\Delta}(X_1; \tilde{X}_1) = & \frac{2}{A(A-1)} \sum_{ij} \sum_{i'j'} \int d(X_2, \dots, X_A) \times \\ & \left[(A-1) \Psi_{[\beta_i\beta_j]}^{\Delta N}(X_1, X_2) \Psi_{(\beta_i\beta_j)^{-1}}^N(X_3, X_4, \dots, X_A) \times \right. \\ & \times \Psi_{[\beta_{i'}\beta_{j'}]}^{\Delta N^*}(\tilde{X}_1, X_2) \Psi_{(\beta_{i'}\beta_{j'})^{-1}}^{N^*}(X_3, X_4, \dots, X_A) + \\ & \frac{(A-1)(A-2)}{2} \Psi_{[\beta_i\beta_j]}^{\Delta N}(X_3, X_2) \Psi_{(\beta_i\beta_j)^{-1}}^N(X_1, X_4, \dots, X_A) \times \\ & \left. \times \Psi_{[\beta_{i'}\beta_{j'}]}^{\Delta N^*}(X_3, X_2) \Psi_{(\beta_{i'}\beta_{j'})^{-1}}^{N^*}(\tilde{X}_1, X_4, \dots, X_A) \right]. \quad (4.5) \end{aligned}$$

The first summand of the formula (4.5) corresponds to the interaction of the photon with ΔN system. We shall mark it $\rho_{\Delta\Delta}^S(X_1; \tilde{X}_1)$. The second summand is $\rho_{\Delta\Delta}^C(X_1; \tilde{X}_1)$, corresponds to the interaction of the photon with the nucleon core. As there is orthogonality of

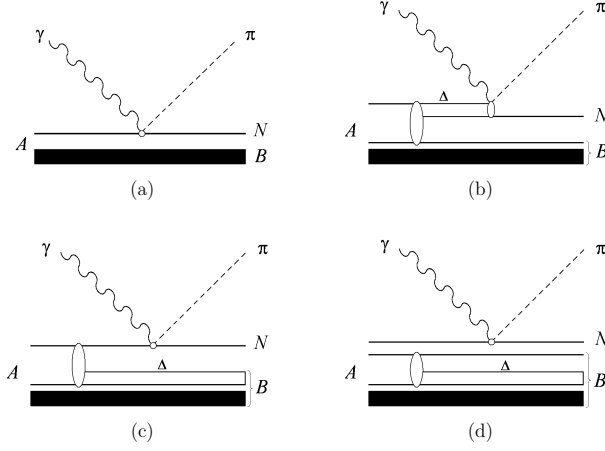


Figure 1: The diagrams illustrating the direct mechanisms of the πN pair photoproduction on nuclei in the $A(\gamma, \pi N)B$ reaction.

one-particle wave function, we shall get the expression for $\rho_{\Delta\Delta}^S(X_1; \tilde{X}_1)$ after the integration over the variable of the particles with number 3, ..., A

$$\rho_{\Delta\Delta}^S(X_1; \tilde{X}_1) = \frac{2}{A} \sum_{ij} \int dX_2 \Psi_{[\beta_i\beta_j]}^{\Delta N}(X_1, X_2) \Psi_{[\beta_i\beta_j]}^{\Delta N*}(\tilde{X}_1, X_2).$$

We will write the expressions for the second summand in (4.5) $\rho_{\Delta\Delta}^C(X_1; \tilde{X}_1)$ as

$$\begin{aligned} \rho_{\Delta\Delta}^C(X_1; \tilde{X}_1) &= \frac{A-2}{A} \sum_{ij} N_{\Delta ij} \int d(X_4, \dots, X_A) \Psi_{(\beta_i\beta_j)-1}^N(X_1, X_4, \dots, X_A) \times \\ &\quad \times \Psi_{(\beta_i\beta_j)-1}^{N*}(\tilde{X}_1, X_4, \dots, X_A), \end{aligned}$$

where $N_{\Delta ij}$ is the norm of wave function $\Psi_{[\beta_i\beta_j]}^{\Delta N}$, satisfying the relationship

$$\sum_{ij} N_{\Delta ij} = N_{\Delta}.$$

Writing the wave function of the nucleon₁ core $\Psi_{(\beta_i\beta_j)-1}^N$ in the form of decompositions

$$\Psi_{(\beta_i\beta_j)-1}^N(X_1, X_4, \dots, X_A) = A_{1;2\dots(A-1)\neq ij}^S \Psi_{\beta_1}(X_1) \Psi_{(\beta_i\beta_j\beta_1)-1}^N(X_4, \dots, X_A)$$

and performing integration over the variables X_4, \dots, X_A , we will get

$$\rho_{\Delta\Delta}^C(X_1; \tilde{X}_1) = \frac{1}{A} \sum_{ij, k \neq ij} N_{\Delta ij} \Psi_{\beta_k}(X_1) \Psi_{\beta_k}^*(\tilde{X}_1).$$

Using (3.5) and (4.2), we will write the summands of the expressions (4.5) as follows

$$\rho_{\Delta\Delta} = \rho_{\Delta\Delta}^{\Delta} + \rho_{\Delta\Delta}^N + \rho_{\Delta\Delta}^C,$$

where

$$\begin{aligned}
\rho_{\Delta\Delta}^{\Delta}(X_1; \tilde{X}_1) &= \phi_{\Delta}(m_1) \left[\frac{1}{A} \int dx_2 \rho^{\Delta N}(x_1, x_2; \tilde{x}_1, x_2) \right] \phi_{\Delta}^*(\tilde{m}_1), \\
\rho_{\Delta\Delta}^N(X_1; \tilde{X}_1) &= \phi_N(m_1) \left[\frac{1}{A} \int dx_2 \rho^{\Delta N}(x_2, x_1; x_2, \tilde{x}_1) \right] \phi_N^*(\tilde{m}_1), \\
\rho_{\Delta\Delta}^C(X_1; \tilde{X}_1) &= \phi_N(m_1) \left[\frac{1}{A} \sum_{ij} N_{\Delta ij} \rho_{(\beta_i \beta_j)^{-1}}^N(x_1, \tilde{x}_1) \right] \phi_N^*(\tilde{m}_1), \\
\rho^{\Delta N}(x_1, x_2; \tilde{x}_1, \tilde{x}_2) &= \sum_{ij} \rho_{[ij]}^{\Delta N}(x_1, x_2; \tilde{x}_1, \tilde{x}_2), \\
\rho_{[ij]}^{\Delta N}(x_1, x_2; \tilde{x}_1, \tilde{x}_2) &= \Psi_{[\beta_i \beta_j]}^{\Delta N}(x_1, x_2) \Psi_{[\beta_i \beta_j]}^{\Delta N*}(\tilde{x}_1, \tilde{x}_2), \\
\rho_{(\beta_i \beta_j)^{-1}}^N(x_1, \tilde{x}_1) &= \sum_{k \neq ij}^A \Psi_{\beta_k}(x_1) \Psi_{\beta_k}^*(\tilde{x}_1).
\end{aligned}$$

In the expression for the density matrix $\rho_{\Delta\Delta}^{\Delta}(X_1; \tilde{X}_1)$ the particle "1" is an isobar. The reaction mechanism corresponding to the $\rho_{\Delta\Delta}^{\Delta}(X_1; \tilde{X}_1)$, is illustrated by the diagram in Fig. 1b. In this case, the production of the pion results from the process $\gamma\Delta \rightarrow N\pi$, under which the virtual isobar taking up photon moves over to the real state and decays on the nucleon and the pion.

In expressions for the density matrix $\rho_{\Delta\Delta}^N(X_1; \tilde{X}_1)$ and $\rho_{\Delta\Delta}^C(X_1; \tilde{X}_1)$ the particle "1" is nucleon. The reaction mechanisms corresponding to the $\rho_{\Delta\Delta}^N(X_1; \tilde{X}_1)$ and $\rho_{\Delta\Delta}^C(X_1; \tilde{X}_1)$ condition of the isobar configurations are illustrated by the diagrams in Fig. 1c and Fig. 1d. They differ by the composition and the condition of the barions, forming the remaining nuclei.

5 Two-particle density matrix

Two-particle density matrix is used in the expressions for the quadrate of the modulus of the exchange amplitude (2.2) for the reaction $A(\gamma, \pi N)B$. For calculation of the density matrix

$$\rho_{NN}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \int d(X_3, \dots, X_A) \Psi_N(X_1, X_2, \dots, X_A) \Psi_N^*(\tilde{X}_1, \tilde{X}_2, \dots, X_A) \quad (5.1)$$

we will present the wave function Ψ_N in the form of the expansion

$$\begin{aligned}
\Psi_N(X_1, X_2, X_3, \dots, X_A) &= \left(\frac{2N_N}{A(A-1)} \right)^{1/2} \sum_{ij} (-1)^{i+j+1} \Psi_{\beta_i \beta_j}^N(X_1, X_2) \times \\
&\times \Psi_{(\beta_i \beta_j)^{-1}}^N(X_3, \dots, X_A),
\end{aligned} \quad (5.2)$$

where

$$\Psi_{\beta_i \beta_j}^N(X_1, X_2) = A_{1;2} \Psi_{\beta_i}(X_1) \Psi_{\beta_j}(X_2).$$

After substituting (5.2) in (5.1), taking into account (4.2), we will get

$$\rho_{NN}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) =$$

$$\phi_{NN}(m_1, m_2) \left[\frac{2N_N}{A(A-1)} \sum_{ij} \Psi_{\beta_i \beta_j}^N(x_1, x_2) \Psi_{\beta_i \beta_j}^{N*}(\tilde{x}_1, \tilde{x}_2) \right] \phi_{NN}^*(\tilde{m}_1, \tilde{m}_2).$$

We shall go to the consideration of the density matrix $\rho_{\Delta\Delta}$

$$\rho_{\Delta\Delta}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \int d(X_3, \dots, X_A) \Psi_{\Delta}(X_1, X_2, \dots, X_A) \Psi_{\Delta}^*(\tilde{X}_1, \tilde{X}_2, \dots, X_A).$$

As a result of transformations of this expression, executed similarly in the previous section, the density matrix $\rho_{\Delta\Delta}$ may be written as

$$\rho_{\Delta\Delta} = \rho_{\Delta\Delta}^{\Delta N} + \rho_{\Delta\Delta}^{N\Delta} + \rho_{\Delta\Delta}^{\Delta C} + \rho_{\Delta\Delta}^{NC} + \rho_{\Delta\Delta}^{CN} + \rho_{\Delta\Delta}^{C\Delta} + \rho_{\Delta\Delta}^{CC},$$

where

$$\rho_{\Delta\Delta}^{\Delta N}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{\Delta N}(m_1, m_2) \left[\frac{1}{A(A-1)} \rho^{\Delta N}(x_1, x_2; \tilde{x}_1, \tilde{x}_2) \right] \phi_{\Delta N}^*(\tilde{m}_1, \tilde{m}_2),$$

$$\rho_{\Delta\Delta}^{N\Delta}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{\Delta N}(m_2, m_1) \left[\frac{1}{A(A-1)} \rho^{\Delta N}(x_2, x_1; \tilde{x}_2, \tilde{x}_1) \right] \phi_{\Delta N}^*(\tilde{m}_2, \tilde{m}_1),$$

$$\rho_{\Delta\Delta}^{\Delta C}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{\Delta N}(m_1, m_2) \left[\frac{1}{A(A-1)} \sum_{ij} \int d(x_3) \rho_{[ij]}^{\Delta N}(x_1, x_3; \tilde{x}_1, x_3) \rho_{(ij)-1}^N(x_2; \tilde{x}_2) \right] \phi_{\Delta N}^*(\tilde{m}_1, \tilde{m}_2),$$

$$\rho_{\Delta\Delta}^{NC}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{NN}(m_1, m_2) \left[\frac{1}{A(A-1)} \sum_{ij} \int d(x_3) \rho_{[ij]}^{\Delta N}(x_3, x_1; x_3, \tilde{x}_1) \rho_{(ij)-1}^N(x_2; \tilde{x}_2) \right] \phi_{NN}^*(\tilde{m}_1, \tilde{m}_2),$$

$$\rho_{\Delta\Delta}^{CN}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{NN}(m_1, m_2) \left[\frac{1}{A(A-1)} \sum_{ij} \int d(x_3) \rho_{[ij]}^{\Delta N}(x_3, x_2; x_3, \tilde{x}_2) \rho_{(ij)-1}^N(x_1; \tilde{x}_1) \right] \phi_{NN}^*(\tilde{m}_1, \tilde{m}_2),$$

$$\rho_{\Delta\Delta}^{C\Delta}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{\Delta N}(m_2, m_1) \left[\frac{1}{A(A-1)} \sum_{ij} \int d(x_3) \rho_{[ij]}^{\Delta N}(x_2, x_3; \tilde{x}_2, x_3) \rho_{(ij)-1}^N(x_1; \tilde{x}_1) \right] \phi_{\Delta N}^*(\tilde{m}_2, \tilde{m}_1),$$

$$\rho_{\Delta\Delta}^{CC}(X_1, X_2; \tilde{X}_1, \tilde{X}_2) = \phi_{NN}(m_1, m_2) \left[\frac{2}{A(A-1)} \sum_{ij} N_{\Delta ij} \rho_{(ij)-1}^{NN}(x_1, x_2; \tilde{x}_1, \tilde{x}_2) \right] \phi_{NN}^*(\tilde{m}_1, \tilde{m}_2),$$

$$\rho_{(ij)-1}^{NN}(x_1, x_2; \tilde{x}_1, \tilde{x}_2) = \sum_{kl \neq ij} \Psi_{\beta_k \beta_l}^N(x_1, x_2) \Psi_{\beta_k \beta_l}^{N*}(\tilde{x}_1, \tilde{x}_2).$$

The exchange amplitudes, corresponding to the matrix $\rho_{\Delta\Delta}^{N\Delta}$ and $\rho_{\Delta\Delta}^{C\Delta}$ are zero, because of the orthogonality of the wave functions $\phi_N(m_2)$ and $\phi_{\Delta}(m_2)$. The remaining six summands ρ_{NN} , $\rho_{\Delta\Delta}^{\Delta N}$, $\rho_{\Delta\Delta}^{\Delta C}$, $\rho_{\Delta\Delta}^{NC}$, $\rho_{\Delta\Delta}^{CN}$ and $\rho_{\Delta\Delta}^{CC}$ of the two-particle density matrix correspond to the mechanism of the reactions in the usual space, which are illustrated by the diagram shown in Fig. 2 in the same order.

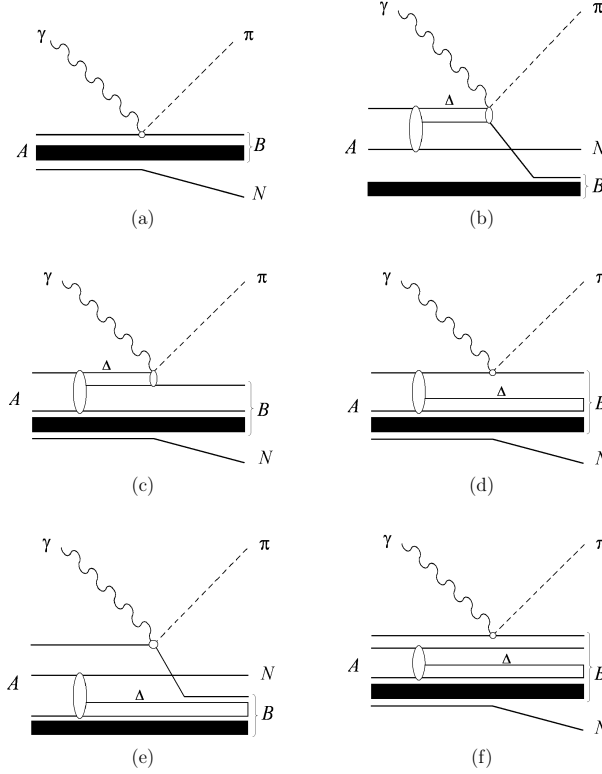


Figure 2: The diagrams illustrating the exchange mechanisms of the πN pair photoproduction on nuclei in the $A(\gamma, \pi N)B$ reaction.

6 Transition operator

With the help of the formula

$$\langle x' | t_{\gamma B \pi} | x \rangle = \sum_{m' m} \phi_N^*(m') \langle X' | t_{\gamma \pi} | X \rangle \phi_B(m),$$

matrix element of the operator $t_{\gamma \pi}$ between the one-particle intrinsic states defines the transition operator $\langle x' | t_{\gamma B \pi} | x \rangle$ in the configuration space, which acts on the space \mathbf{r} , spin s , and isospin t coordinates.

Using the S -matrix approach for the description of an elementary process $\gamma B \rightarrow N \pi$, we will find the transition operator $t_{\gamma B \pi}$. We will suppose that S -matrix has the standard expansion in power of the interaction Lagrangian, in which the strong interaction fields of the nucleon, pion and isobar, the vector potential of photon are the operators acting on the space \mathbf{r} , spin s , and isospin t coordinates. We will write the Lagrangian of the strong interaction barion fields and the pion field as $L_s = \mathbf{j}_\pi(\mathbf{r}, t) \phi(\mathbf{r}, t)$, where $\mathbf{j}_\pi(\mathbf{r}, t)$ is the pion current. The Lagrangian of the electromagnetic interaction may be written as $L_\gamma = j_\mu(\mathbf{r}, t) A^\mu(\mathbf{r}, t)$, where $j_\mu(\mathbf{r}, t)$ is the electromagnetic current. Then, the transition operator $t_{\gamma B \pi}$ may be written as

$$t_{\gamma B \pi} = \varphi_a \mathbf{t}_{\gamma B \pi}^\mu \epsilon_\mu^\lambda.$$

Here ϵ^λ is 4-vector of the photon polarization; φ_a is the covariant unit vector of the cyclical basis describing the isotopic state of the pion; index a takes on the values $+$, 0 , $-$, which fit

with the positive, neutral and negative pions;

$$\mathbf{t}_{\gamma B\pi}^\mu = \int d\mathbf{r} d\mathbf{r}' e^{i(\mathbf{p}_\gamma \cdot \mathbf{r} - \mathbf{p}_\pi \cdot \mathbf{r}')} \int dt e^{-iE_\gamma t} T(j_\mu(\mathbf{r}, t) \mathbf{j}_\pi(\mathbf{r}', 0))|_{B \rightarrow N}, \quad (6.1)$$

where index $B \rightarrow N$ under T -product of the currents designates that it necessarily leaves those components of the electromagnetic and pion currents, which give rise to the $N \rightarrow N$ and the $\Delta \rightarrow N$ transitions. The expression of the current comes from the interaction Lagrangian constructing from the photon, nucleon, Δ -isobar and pion fields. Using the operator (6.1), we will go to the momentum representation of the matrix element $\langle x' | \mathbf{t}_{\gamma B\pi}^\mu | x \rangle$ and insert the full set of the intermediate single-particle barion and meson states with the minimal masses of the barions and the pions. As a result, we will obtain

$$\begin{aligned} \langle x' | \mathbf{t}_{\gamma B\pi}^\mu | x \rangle &= \int d\mathbf{p}_B e^{i\mathbf{p}_N \cdot \mathbf{r}'} \sum_{m_{\sigma_N}, m_{\tau_N}, m_{\sigma_B}, m_{\tau_B}} \xi_{m_{\sigma_N}, m_{\tau_N}}^*(s', t') \times \\ &\langle m_{\sigma_N}, m_{\tau_N} | \mathbf{T}_{\gamma B \rightarrow N\pi}^\mu(\mathbf{p}_N, \mathbf{p}_B) | m_{\sigma_B}, m_{\tau_B} \rangle \xi_{m_{\sigma_B}, m_{\tau_B}}(s, t) e^{-i\mathbf{p}_B \cdot \mathbf{r}}. \end{aligned}$$

Here B and N are the indices of the initial barion and the final nucleon; \mathbf{p}_B and \mathbf{p}_N are the momenta of the barion and the nucleon for the process $\gamma B \rightarrow \pi N$, $\mathbf{p}_N = \mathbf{p}_B + \mathbf{q}$, where $\mathbf{q} = \mathbf{p}_\gamma - \mathbf{p}_\pi$ is the transfer momentum, $\xi_{m_{\sigma_N}, m_{\tau_N}}$ and $\xi_{m_{\sigma_B}, m_{\tau_B}}$ are the spin-isospin wave functions of the nucleon and the barion. If it designates: $\alpha \equiv \mathbf{p}, m_\sigma, m_\tau$ is the barion state index; n is the index of the intermediate barion, α_{π^a} is the index of the pion, then

$$\begin{aligned} \langle m_{\sigma_N}, m_{\tau_N} | \mathbf{T}_{\gamma B \rightarrow N\pi}^\mu(\mathbf{p}_N, \mathbf{p}_B) | m_{\sigma_B}, m_{\tau_B} \rangle &= \\ \frac{(2\pi)^3}{i} \left[\sum_{n=N, \Delta} \frac{\langle \alpha_N | j^\mu(0, 0) | \alpha_n \rangle \langle \alpha_n | \mathbf{j}_\pi(0, 0) | \alpha_B \rangle}{E_\gamma + E_n - E_N - i\varepsilon} + \right. \\ &+ \sum_{\tilde{n}=N, \Delta} \frac{\langle \alpha_N | \mathbf{j}_\pi(0, 0) | \alpha_{\tilde{n}} \rangle \langle \alpha_{\tilde{n}} | j^\mu(0, 0) | \alpha_B \rangle}{E_{\tilde{n}} - E_\gamma - E_B - i\varepsilon} - \\ &\left. - 2 \frac{E_\pi^2}{E_\pi} \sum_b \frac{\langle \alpha_{\pi^a} | j^\mu(0, 0) | \alpha_{\tilde{\pi}^b} \rangle \langle \alpha_N | \mathbf{j}_\pi(0, 0) | \alpha_B \rangle}{E_\pi^2 - \mathbf{p}_\pi^2 - m_\pi^2} + i \langle \alpha_N | \mathbf{j}_{\gamma\pi NB}^\mu | \alpha_B \rangle \right], \end{aligned}$$

where $\mathbf{p}_n = \mathbf{p}_B - \mathbf{p}_\pi$, $\mathbf{p}_{\tilde{n}} = \mathbf{p}_\gamma + \mathbf{p}_B$ and $\mathbf{p}_{\tilde{\pi}} = -\mathbf{q}$ are the momenta of the intermediate barions and mesons, $E_n = (\mathbf{p}_n^2 + m_n^2)^{1/2}$, $E_{\tilde{n}} = (\mathbf{p}_{\tilde{n}}^2 + m_{\tilde{n}}^2)^{1/2}$, $E_{\tilde{\pi}} = E_\gamma - E_\pi$.

The matrix elements of the currents are written by means of the nonrelativistic currents as

$$\begin{aligned} \langle \alpha_2 | j^\mu(0, 0) | \alpha_1 \rangle &= \frac{1}{2} (E_2 E_1)^{-1/2} \xi_{m_{\sigma_2}, m_{\tau_2}}^* j_\mu^{B_2 \leftarrow B_1}(\mathbf{p}_2, \mathbf{p}_1) \xi_{m_{\sigma_1}, m_{\tau_1}}, \\ \langle \alpha_2 | \mathbf{j}_\pi(0, 0) | \alpha_1 \rangle &= \frac{1}{2} (E_2 E_1)^{-1/2} \xi_{m_{\sigma_2}, m_{\tau_2}}^* \mathbf{j}_\pi^{B_2 \leftarrow B_1}(\mathbf{p}_2, \mathbf{p}_1) \xi_{m_{\sigma_1}, m_{\tau_1}}, \\ \langle \alpha_{\pi^a} | \mathbf{j}_\mu(0, 0) | \alpha_{\tilde{\pi}^b} \rangle &= \frac{1}{2} (E_\pi E_{\tilde{\pi}})^{-1/2} \varphi_a^* j_\mu^{\pi \leftarrow \pi}(\mathbf{p}_\pi, \mathbf{p}_{\tilde{\pi}}) \varphi_b. \end{aligned}$$

The explicit expressions of nonrelativistic currents are

$$\mathbf{j}_\gamma^{N \leftarrow N}(\mathbf{p}_2, \mathbf{p}_1) = \frac{e}{2m_N} \left[(\mathbf{p}_2 + \mathbf{p}_1) \frac{1 + \tau_3}{2} + i\boldsymbol{\sigma} \times (\mathbf{p}_2 - \mathbf{p}_1) \left(\frac{1 + \tau_3}{2} \mu_p + \frac{1 - \tau_3}{2} \mu_n \right) \right],$$

$$\begin{aligned}
\mathbf{j}_\gamma^{N\leftarrow\Delta}(\mathbf{p}_2, \mathbf{p}_1) &= \frac{e}{2m_\Delta} \mu_{N\Delta} [(\mathbf{p}_2 - \mathbf{p}_1) \times \mathbf{S}^+] \mathbf{T}^+, \\
\mathbf{j}_\gamma^{\Delta\leftarrow\Delta}(\mathbf{p}_2, \mathbf{p}_1) &= \frac{e}{2m_\Delta} \left[(\mathbf{p}_2 + \mathbf{p}_1) \frac{1 + \Theta_3}{2} + i\boldsymbol{\Sigma} \times (\mathbf{p}_2 - \mathbf{p}_1) \frac{1 + \Theta_3}{2} \mu_\Delta \right], \\
\mathbf{j}_\gamma^{\pi N\leftarrow\Delta} &= ie \frac{f_{\pi N\Delta}}{m_\pi} \mathbf{S}^+ \mathbf{T}^+, \quad \mathbf{j}_\gamma^{\pi N\leftarrow N} = ie \frac{f_{\pi NN}}{m_\pi} \boldsymbol{\sigma} \boldsymbol{\tau}, \\
\mathbf{j}_\gamma^{\pi\leftarrow\pi}(\mathbf{p}_2, \mathbf{p}_1) &= -ie (\mathbf{p}_2 - \mathbf{p}_1), \quad \mathbf{j}_\pi^{\Delta\leftarrow\Delta}(\mathbf{p}_2, \mathbf{p}_1) = i \frac{f_{\pi\Delta\Delta}}{m_\pi} \boldsymbol{\Sigma} (\mathbf{p}_2 - \mathbf{p}_1) \mathbf{T}_\Delta, \\
\mathbf{j}_\pi^{N\leftarrow N}(\mathbf{p}_2, \mathbf{p}_1) &= i \frac{f_{\pi NN}}{m_\pi} \boldsymbol{\sigma} (\mathbf{p}_2 - \mathbf{p}_1) \boldsymbol{\tau}, \quad \mathbf{j}_\pi^{N\leftarrow\Delta}(\mathbf{p}_2, \mathbf{p}_1) = i \frac{f_{\pi N\Delta}}{m_\pi} \mathbf{S}^+ (\mathbf{p}_2 - \mathbf{p}_1) \mathbf{T}^+.
\end{aligned}$$

Here $\boldsymbol{\Sigma}$ and \mathbf{T}_Δ are the analogues of the Pauli matrixes for objects with spin and isospin 3/2; Θ_3 is matrix of the charge for particles with isospin 3/2. The vector and scalar magnetic moments of the nucleon and the magnetic moments of the isobar are accordingly $\mu_V = 1.8$, $\mu_S = -0.06$ and $\mu_\Delta = 4.3$, $\mu_{\gamma N\Delta} = 3.42$ in terms of nuclear magnetons. The hadronic coupling constants are $f_{\pi NN}^2/4\pi = 0.08$, $f_{\pi N\Delta} = 2.836$, $f_{\pi\Delta\Delta} = 4 f_{\pi NN}/5$.

The result for $\langle m_{\sigma_N}, m_{\tau_N} | \mathbf{T}_{\gamma B \rightarrow N\pi}^\mu(\mathbf{p}_N, \mathbf{p}_B) | m_{\sigma_B}, m_{\tau_B} \rangle$ answers, in general, the non-locale interaction that brings about the need of the calculation of multivariate integral on $\mathbf{p}_B, \mathbf{r}', \mathbf{r}$. Supposing the momentum \mathbf{p}_N in the operator $\mathbf{T}_{\gamma B \rightarrow N\pi}^\mu(\mathbf{p}_N, \mathbf{p}_B)$ is fixed $\mathbf{p}_N = \tilde{\mathbf{p}}$ gives the local transition operator $\langle x' | t_{\gamma B\pi}^\mu | x \rangle$

$$\langle x' | t_{\gamma B\pi}^\mu | x \rangle = \delta(\mathbf{r}' - \mathbf{r}) e^{i(\mathbf{p}_\gamma \mathbf{r} - \mathbf{p}_\pi \mathbf{r}')} \langle s', t' | T_{\gamma B\pi}^\mu(\tilde{\mathbf{p}}, \tilde{\mathbf{p}} - \mathbf{q}) | s, t \rangle.$$

Here

$$\begin{aligned}
&\langle s', t' | \mathbf{T}_{\gamma B\pi}^\mu(\tilde{\mathbf{p}}, \tilde{\mathbf{p}} - \mathbf{q}) | s, t \rangle = \\
&\sum_{m_{\sigma_N}, m_{\tau_N}, m_{\sigma_B}, m_{\tau_B}} \zeta_{m_{\sigma_N}, m_{\tau_N}}^*(s', t') \langle m_{\sigma_N}, m_{\tau_N} | \mathbf{T}_{\gamma B \rightarrow N\pi}^\mu(\tilde{\mathbf{p}}, \tilde{\mathbf{p}} - \mathbf{q}) | m_{\sigma_B}, m_{\tau_B} \rangle \zeta_{m_{\sigma_B}, m_{\tau_B}}(s, t).
\end{aligned}$$

At calculating of the amplitude of the transition $\gamma B \rightarrow N\pi$, as momentum $\tilde{\mathbf{p}}$, we shall take the momentum, which canonically conjugate to the coordinate \mathbf{r} in the nucleon wave function. For the amplitude, corresponding to the diagram on fig. 1a, the momentum $\tilde{\mathbf{p}}$ is a momentum \mathbf{p}_n of the free nucleon. In the exchange amplitude the momentum $\tilde{\mathbf{p}}$ is the integration variable.

Using the explicit form of the expression of non-relativistic currets, the transition operator $t_{\gamma\Delta\pi}$ may be written as

$$t_{\gamma\Delta\pi} = \varphi_a \sum_{i=1}^4 \sum_{j=1}^3 f_{ij} M_i \mathbf{I}_j,$$

where M_i are the independent spin structures

$$\begin{aligned}
M_1 &= \boldsymbol{\varepsilon}^\lambda \cdot \mathbf{S}^+; & M_2 &= i\boldsymbol{\sigma} \cdot [\mathbf{p}_\gamma \times \boldsymbol{\varepsilon}^\lambda] \mathbf{p}_\pi \cdot \mathbf{S}^+; \\
M_3 &= \mathbf{p}_\pi \cdot \boldsymbol{\varepsilon}^\lambda \mathbf{p}_\gamma \cdot \mathbf{S}^+; & M_4 &= \mathbf{p}_\pi \cdot \boldsymbol{\varepsilon}^\lambda \mathbf{p}_\pi \cdot \mathbf{S}^+.
\end{aligned}$$

Here $\boldsymbol{\varepsilon}^\lambda$ is 3-vector of the photon polarization.

The isospin structures are

$$\mathbf{I}_1 = \mathbf{T}^+, \quad \mathbf{I}_2 = \boldsymbol{\tau} T_3^+, \quad \mathbf{I}_3 = \tau_3 \mathbf{T}^+$$

The values f_{ij} are

$$\begin{aligned}
f_{11} &= \alpha \frac{\mu_\Delta}{3a} \mathbf{p}_\pi \mathbf{p}_\gamma, & f_{12} &= \alpha \left[\frac{2\mu_\Delta}{3a} + \frac{F}{4b} \right] \mathbf{p}_\pi \mathbf{p}_\gamma, & f_{13} &= \alpha \left[\frac{\mu_\Delta}{a} + \frac{F}{6b} \right] \mathbf{p}_\pi \mathbf{p}_\gamma, \\
f_{21} &= \alpha \left[\frac{\mu_\Delta}{6a} + \frac{(1+\mu_S)}{2b} \right], & f_{22} &= \alpha \left[-\frac{\mu_\Delta}{3a} + \frac{3F}{4b} \right], & f_{23} &= \alpha \left[-\frac{\mu_\Delta}{3a} - \frac{F}{2b} + \frac{(1+\mu_S)}{2c} \right], \\
f_{31} &= -\alpha \left[\frac{\mu_\Delta}{3a} + \frac{4m_N}{d} \right], & f_{32} &= -\alpha \left[-\frac{2\mu_\Delta}{3a} + \frac{F}{4b} \right], & f_{33} &= \alpha \left[-\frac{\mu_\Delta}{a} + \frac{1}{6b} \right], \\
f_{41} &= \alpha \frac{4m_N}{d}, & f_{42} &= 0, & f_{43} &= 0.
\end{aligned}$$

Here

$$\begin{aligned}
F &= \mu_{\gamma N \Delta} \frac{f_{\pi \Delta \Delta}}{f_{\pi N \Delta}}, & \alpha &= i \frac{e}{2m_N} \frac{f_{\pi N \Delta}}{m_\pi}, \\
a &= E_\Delta + E_\gamma - E_a + \frac{i}{2} \Gamma_\Delta, & b &= E_\Delta - E_\pi - E_b + i\varepsilon, \\
c &= E_\Delta - E_\pi - E_c + i\varepsilon, & d &= (\mathbf{p}_N - \mathbf{p}_\Delta)^2 - (E_N - E_\Delta)^2 + m_\pi^2 + i\varepsilon, \\
E_a &= ((\mathbf{p}_\Delta + \mathbf{p}_\gamma)^2 + m_\Delta^2)^{1/2}, & E_b &= ((\mathbf{p}_\Delta - \mathbf{p}_\pi)^2 + m_\Delta^2)^{1/2}, & E_c &= ((\mathbf{p}_\Delta - \mathbf{p}_\pi)^2 + m_N^2)^{1/2}.
\end{aligned}$$

We shall use the non-relativistic operator of Blomqvist-Laget [20] as the transition operator $t_{\gamma N \pi}$.

7 Cross section of the $A(\gamma, \pi N)B$ reaction

Following from the consideration of one- and two-particle density matrixes, an account of the isobar configurations in the atomic nuclei results in a significant increase of possible direct and exchange mechanisms of the reactions in the usual space. One-particle and two-particle density matrixes present themselves as some combinations of one-particle wave functions of the nucleons bound in nuclei $\Psi_{\beta_i}(x)$ and wave function $\Psi_{[\beta_i \beta_j]}^{\Delta N}(x_1, x_2)$, describing the ΔN system. Separate components of one-particle and two-particle density matrixes are connected with different mechanisms of considered reaction. For qualitative estimation of the kinematic area, where different mechanisms of the reactions are shown, the momentum distributions of the isobar ρ^Δ and proton ρ^N of nucleus ^{12}C defined as

$$\begin{aligned}
\rho^N(p) &= \sum_i \int dy_N \Psi_{\beta_i}^*(y_N) \delta(p - p_N) \delta_{1/2, m_{\tau_i}} \Psi_{\beta_i}(y_N) / p^2, \\
\rho^\Delta(p) &= \sum_{ij} \int d(y_\Lambda, y_N) \Psi_{[\beta_i \beta_j]}^{\Delta N*}(y_\Delta, y_N) \delta(p - p_\Delta) \Psi_{[\beta_i \beta_j]}^{\Delta N}(y_\Delta, y_N) / p^2
\end{aligned}$$

are given in Fig. 3, where $y \equiv \mathbf{p}, s, t$; $\Psi_{\beta_i}(y_N)$ and $\Psi_{[\beta_i \beta_j]}^{\Delta N}(y_\Lambda, y_N)$ are the Fourier transforms of the wave functions $\Psi_{\beta_i}(x_N)$ and $\Psi_{[\beta_i \beta_j]}^{\Delta N}(x_\Lambda, x_N)$.

Let us consider the direct mechanisms of the reaction. First of all it is necessary to note that all four mechanisms of the reaction shown in Fig. 1(a-d), give contributions to the cross section of the photoproduction of the π^+n , π^-p , π^0p and π^0n pairs. The contributions to the cross section of the photoproduction of the π^+p and π^-n pairs are due to the mechanism in

Fig. 1b. So, these two reactions are perspective for the study of the virtual states of the isobar in a nucleus. Since the density of the momentum distribution of the nucleon under small momentum is by several orders more than the density of the momentum distribution of the Δ -isobar, the mechanisms of the reactions corresponding to the diagrams in Fig. 1a and Fig. 1d practically completely define the behavior of the cross section of reactions in this kinematic area. At large momentum transfer to the nucleus exceeding 400 MeV/c, the mechanisms of the reactions corresponding to the diagrams in Fig. 1b and in Fig. 1c dominate. The relative contribution of these diagrams is defined, basically, by the probability of the $\gamma N \rightarrow N\pi$ and $\gamma\Delta \rightarrow N\pi$ transitions. The direct mechanism of a pion photoproduction corresponding to the diagrams in Fig. 1a and taking into account only the nucleon configurations is analysed in detail in Refs. [21–23].

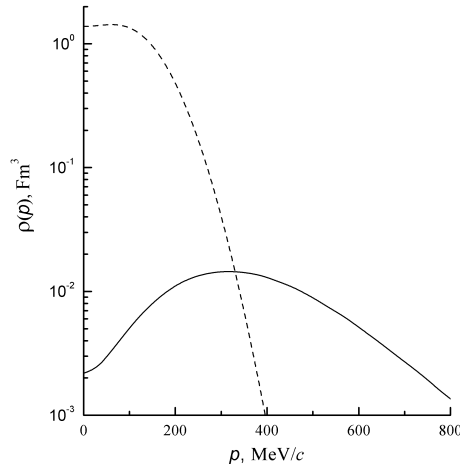


Figure 3: The momentum distributions of the Δ -isobar (solid curve) and protons (dashed curve) of nucleus ^{12}C

The exchange mechanisms of the reactions shown in Fig. 2(a–f) may be divided into two groups. One group includes the mechanisms in which the nucleon belonging to the nucleon core of the nucleus becomes free. Manifestations of the exchange mechanism of the neutral pion photoproduction within the framework of the model, taking into account only the nucleon configurations of the nuclei, were analysed in the works [24, 25]. The contribution of the appropriate exchange transition amplitudes quickly decreases with growing of the nucleon energy and concentrates near the kinematic area of the coherent pion photoproduction on the residual nucleus, in range of small momentum transfer $\mathbf{p}_\gamma - \mathbf{p}_\pi$. Another group contains mechanisms of the reactions in which the nucleon of the ΔN systems goes to free state. These mechanisms of the $\pi^+ p$ pair photoproduction were considered in the work [26]. The contribution of them to the cross section of the reactions concentrated in the considerably greater range of the nucleon momentum, practically coinciding with range of definition of the virtual isobar momentum distribution.

The cross section corresponding to the different pion-nucleon pair production mechanisms is numerically estimated in respect of $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ and $^{12}\text{C}(\gamma, \pi^+ p)^{11}\text{Be}$ reactions. The differential cross sections are calculated for that mechanisms in which the proton momentum in the final states can be large enough for the experimental checking of the model predictions by means of simultaneous registration of the pion and the proton in the experiment. These are, first of all the direct mechanisms of the reactions and the exchange mechanisms in which

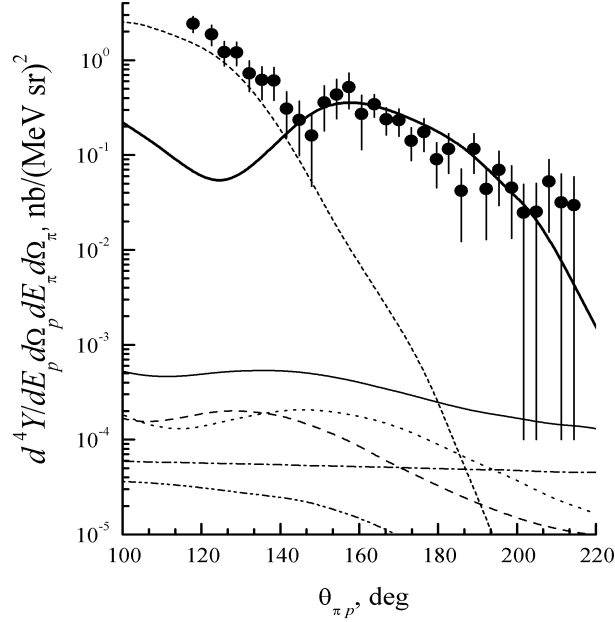


Figure 4: Differential yield of the $^{12}\text{C}(\gamma, \pi^- p)$ reaction versus the opening angle $\theta_{\pi p}$. Thick solid curve: model [11], in which the existence of quasibound isobar-nucleus states in the intermediate state is assumed, short dashed line: quasifree pion photoproduction, dashed and dotted curves: direct mechanisms of the reactions, corresponding to diagrams in Fig. 1b and Fig. 1c, dashed-dotted curves with one and two points: the exchange mechanisms of the reactions, corresponding to the diagrams in Fig. 2e and Fig. 2b, thin solid curve: the total contribution to the cross section of the isobar configurations, data from Ref. [10].

the nucleon of the ΔN systems becomes free.

The cross section of the reaction $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ is calculated in the kinematic area in which the experimental data [10] were interpreted [11] as the manifestation of a quasibound isobar-nucleus states. Figure 4 shows the dependence of the differential reaction yield $d^4Y/dE_p d\Omega_p dE_\pi d\Omega_\pi$ as the function of the opening angle $\theta_{\pi p} = \theta_\pi + \theta_p$, where θ_π and θ_p are the pion and proton polar angles, connected with differential cross section

$$\frac{d^3\sigma}{dE_p d\Omega_p d\Omega_\pi} = (2\pi)^{-5} \frac{p_\pi^3 p_p E_p E_R}{4E_\gamma |E_R p_\pi^2 - E_\pi p_\pi p_R|} \overline{|T_{fi}|^2}$$

by the relation

$$\frac{d^4Y}{dE_p d\Omega_p dE_\pi d\Omega_\pi} = \frac{d^3\sigma}{dE_p d\Omega_p d\Omega_\pi} f(E_\gamma) \left| \frac{\partial E_\gamma}{\partial E_\pi} \right|.$$

Here $f(E_\gamma)$ is the bremsstrahlung spectrum, normalized as

$$\int dE_\gamma f(E_\gamma) E_\gamma = E_{\text{max}},$$

where E_{max} is the maximal energy of the bremsstrahlung, $\overline{|T_{fi}|^2}$ is the quadrate of the modulus of the reaction amplitude T_{fi} , averaged over photon polarization states and summed over proton and residual nucleus states and which is connected with the matrix element of the T_{fi} in (2.1) by the relation

$$T_{fi} = (2\pi)^3 \delta(\mathbf{p}_\gamma - \mathbf{p}_\pi - \mathbf{p}_p - \mathbf{p}_R) T_{fi}.$$

The thick solid curve in Fig. 4 indicates the angular dependence of the differential reaction yield calculated within the framework of the model [11], in which the existence of quasibound isobar-nucleus states in the intermediate state is assumed. The short dashed line in Fig. 4 indicates the quasifree photoproduction of the pions dominant under the small momentum of the residual nucleus which includes the contributions from two mechanisms of the reactions, corresponding to the diagrams in Fig. 1*a* and Fig. 1*d*. The dashed and dotted curves present the contributions to the cross section of the two direct mechanisms of the reactions, corresponding to diagrams in Fig. 1*b* and Fig. 1*c*, in which the product of the pions results from the interaction of the photon with the isobar and the nucleon of the ΔN system.

The contributions of the exchange mechanisms of the reactions to the cross section, corresponding to the diagrams in Fig. 2*e* and Fig. 2*b* are presented by the dashed–dotted curves with one and two points. The thin solid curve in Fig. 4 shows the total contribution to the cross section of the isobar configurations. The final state interaction was taken into account within the framework of the optical model. We see that the angular dependences of the cross sections from some mechanisms of the pion-nucleon pairs production correspond to the experimental large opening angle data. However, the absolute value of the mechanism contributions to the reactions, conditioned by the isobar configurations, is by several orders less. Thus, the behavior of the experimental yield of the $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ reaction for the large opening angles observed in the experiment [10] is impossible to be explained by the effect of isobar configurations in the nucleus ground state.

At present the statistically provided experimental data of the $A(\gamma, \pi^+ p)B$ reaction in range of the large momentum transfer to the residual nucleus are absent. For comparison of the predictions of the presented photoproduction model with the results of the measurements we use the experimental data of the work [27], in which the cross section of the $^{12}\text{C}(\gamma, \pi^+ p)$ reaction, averaged in the kinematic area with mean value of the residual nucleus momentum equal to $\sim 300 \text{ MeV}/c$, is measured.

Figure 5 displays the results of the calculated cross section plotted against the kinetic energy of the proton together with the data of the work [27]. The dashed and dashed-dotted curves present the cross section contributions of the direct and exchange reaction mechanisms, corresponding to the diagram in Fig. 1*b* and Fig. 2*e*. The contribution of the exchange mechanism to the reaction, corresponding to the diagram in Fig. 2*b*, turned out to be less than $10^{-2} \text{ nb/MeV sr}^2$. According to the used model the probability of the internal excitation of the nucleon in the nucleus ^{12}C , as the result of NN -interactions, is ~ 0.01 . As it can be seen in Fig. 5, in spite of low probability of the transition $N \rightarrow \Delta$, the manifestations of isobar configurations in the nucleus ground state allow to explain essential part of the observed cross section of the reaction $^{12}\text{C}(\gamma, \pi^+ p)^{11}\text{Be}$. One of possible explanations of the excess of the experimental cross section over the calculated that is connected with the contribution to the experimental data of the $^{12}\text{C}(\gamma, \pi^+ p)n^{10}\text{Be}$ process, which is realized as a result of the direct knockout of the correlated $\Delta^{++}n$ and Δ^+p pairs by photon.

8 Conclusion

We considered the production of the pion-nucleon pairs when the high energy photon interacted with the nucleus. We used the model in which the nucleus contains excited states of the nucleons – the virtual isobars along with nucleons. The wave function of the Δ -isobar

configuration in the closed shell nuclei was obtained in the harmonic oscillator model of the nuclei with the ls - and jj - coupling. The wave function of the ΔN system state was obtained by means of the solution of the Schrodinger equation. The transition potential with π - and ρ - exchange was used.

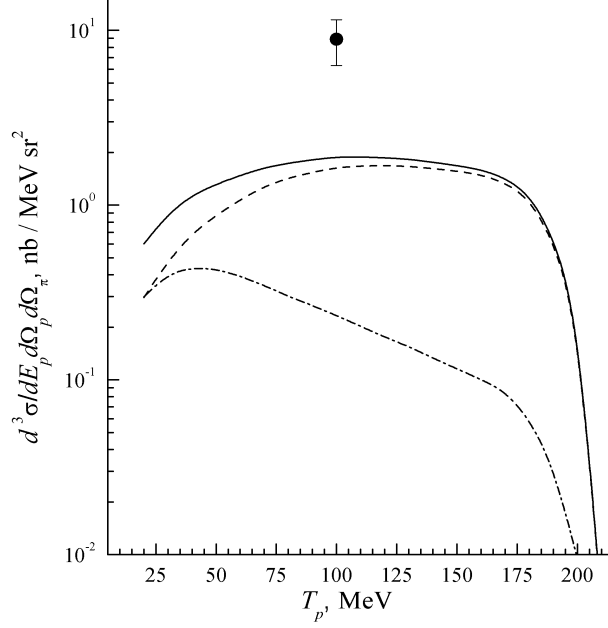


Figure 5: Differential cross section of the $^{12}\text{C}(\gamma, \pi^+ p)$ reaction versus the kinetic energy of the proton T_p . Dashed curve: direct reaction mechanism, corresponding to the diagram in Fig. 1b, dashed-dotted curve: exchange reaction mechanism, corresponding to the diagram in Fig. 2e, solid curve: the total cross section, data from Ref. [27].

Using the S -matrix approach, one-particle operator of $\gamma\Delta \rightarrow \pi N$ transition was found. The S -matrix has been written as the standard expansion in the power of the interaction Lagrangian neglecting terms above the second order. We have taken into account only the Lagrangian of the strong interaction of the nucleon, isobar and the pion fields and Lagrangian of the electromagnetic interaction. At determination of the transition operator, we took into account in s -, t - and u -channels of T -product of the currents the contributions from the intermediate single-particle states with smallest mass – a pion, a nucleon and a $\Delta(1232)$ -isobar.

The analysis of the nucleus density matrixes of the $A(\gamma, \pi N)B$ process was made. Direct and exchange mechanisms of pion-nucleon pairs photoproduction which result from one-particle and two-particle density matrix were considered. The description of the nucleus as a system including alongside with the nucleons their excitation states brought about the significant increase of the possible reaction mechanisms set.

We performed calculations of the differential cross section of the $^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ and $^{12}\text{C}(\gamma, \pi^+ p)^{11}\text{Be}$ reactions. The numerical estimations of the cross section value are made for mechanisms of the reactions, in which the final free nucleon can have the large enough momentum, neglecting the exchange mechanisms, in which the nucleon goes to the free state from state that is lower the Fermi level. In the range of the pion and proton opening angle close to 180° the total contribution of the isobar configuration cross section of the

$^{12}\text{C}(\gamma, \pi^- p)^{11}\text{C}$ reaction is smaller than the cross section observed in experiment [10] of about two order of magnitude. The calculated cross section of the $^{12}\text{C}(\gamma, \pi^+ p)^{11}\text{Be}$ reaction was by about four times smaller than the experimental cross section of the work [27].

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